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Abstract. The interaction of highly resonant interactions can significantly improve the functionality of many electromagnetic devices. However, resonances can be attenuated by Joule and radiation losses. Although in many cases Joule loss can be minimized through the choice of constituent materials, the management of radiation loss is often a big problem. Recent solutions include the use of coupled radiant and subradiant modes, producing narrow asymmetric Fano resonances in a wide range of systems, from defect states in photonic crystals and optical waveguides with mesoscopic ring resonators to nanoscale plasmonic and metamagnetic systems exhibiting interference effects similar to electromagnetic induced transparency. Here we theoretically demonstrate and experimentally confirm a new mechanism of resonant electromagnetic transparency, which gives very narrow isolated symmetric Lorentz lines in toroidal metamaterials.

Keywords: optics, optical waveguide, toroidal metamaterials

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Introduction

The toroidal dipole (and the higher toroidal multipoles) is not part of the standard multipole expansion, which is usually represented as the sum of contributions from two families of common sources of elemental radiation, namely magnetic and electric dynamic multipoles. In a spherical coordinate system, magnetic multipoles are determined by the transverse components of the oscillating current density (for example, currents flowing on the surface of a sphere), while electric multipoles refer to the vibrational charge density.

In nuclear physics, the static toroidal moment is an established concept, also called anapole. It was calculated for a number of nuclear systems and studied in detail theoretically and experimentally, in particular, for its role in nuclear spin-dependent atomic parity violation. The toroidal moment was also identified by the first fundamental calculations in carbon macromolecules and ferroelectric nanostructures.

Review

In the article [1], we looked at non-clipping and masking problems using the principle of surface equivalence, imposing on any arbitrary boundary a gap-tolerance control between the general object (with or without cloak) and the background. After careful demonstration, we apply this model to the irreversible problem, addressing the analog mode and metamolecules, and the masking problem, referring to the non-Foster metaphor design. A direct analytical condition for controlling the scattering of a dielectric object over a surface of interest is obtained. Previous quasi-static results are confirmed and a general closed solution outside the subwavelength regime is presented. In addition, this formulation can be extended to other wave phenomena once the correct tolerance function has been determined (thermal, acoustic, elastomechanical, etc.).

The article [2] investigated the features of light scattering from subwavelength particles made of materials with a high refractive index, caused by the coexistence of separate anapole modes of both electrical and magnetic nature. The similarities and differences of such anapole modes are discussed in detail.

We also show that these two types of anapole modes can be supported simultaneously by subwavelength high-index spherical dielectric particles.

Nonradical charge-current configurations involving toroidal dipoles have attracted attention in a wide range of different fields of physics [3]. In such regime configurations, electric and toroidal dipole modes, the so-called anapoles, cancel each other in the far field [4–6] as a result of both dipole modes sharing the same directional pattern. Such anapoles were first discovered experimentally in configurations involving several particles with a magnetic dipole moment, such as split-ring resonators [4]. Interestingly, it has recently been predicted that these modes can exist in the much simpler case of a single isolated particle with a high index [5, 6]. This prediction was experimentally confirmed at optical frequencies using silicon nanodisks [5]. Excitation of anapole modes in small particles with a high refractive index makes it possible to sharply reduce their scattering cross section, which can reach values much lower than the Rayleigh limit for nonresonant excitation of one electric dipole regime [2]. Until now, anapole modes have been discussed in the context of suppression of electric dipole radiation, that is, as an interference effect due to the simultaneous excitation of electric dipole and electric toroidal dipole moments. An electric dipole is associated with vibrational charges, and a toroidal dipole is associated with a poloidal current on the surface of the torus. However, one can imagine a dual situation in which the magnetizing currents flowing on the torus surface generate a magnetic toroidal dipole mode. Under some circumstances, this magnetic toroidal moment, which would share the characteristics of radiation with a conventional magnetic dipole, could also lead to a complete suppression of radiation from the system (the so-called magnetic anapole).

In [6], studying the scattering of normally incident plane waves by one nanowire, the irreplaceable role of toroidal multipole excitation in multipole expansions of emitting sources was revealed. It is found that for p-for forthreesovanth and s-forthsovanth incident waves, toroidal dipoles can to be efficiently excited in homogeneous dielectric nanowires in the optical spectrum mode. Next, we will demonstrate that plasmonic-core-sheathed nanowires can be invisible through the destructive interference of electric and toroidal dipoles, which could inspire many studies on nanowire-based light interactions and incubate biological and medical applications that require non-invasive detection and measurement.

In [7] it was found that an isotropic homogeneous subwavelength particle with a high refractive index can produce ultra-small total scattering. This effect, which follows from the deceleration of the electric dipole radiation, can be identified as the Fano resonance in the scattering efficiency and is associated with the excitation of the anapole mode in the particle. This anapole mode is non-radiative and comes out of the destructive interference of electric and toroidal dipoles. The invisibility effect can be useful for the design of highly transparent optical materials.

Theoretical methods and materials

In previous articles by the Authors, an adequate expression was obtained for the density of dark matter:

$$B = \int_0^\infty \frac{\xi^2 d\xi}{\sqrt{\eta + \xi^2}} \frac{1 - e^{-\Phi(\eta,\xi)}}{e^{\sigma(\eta+\xi^2)^{1/2}} - 1}, \tag{1}$$

where σ is the darkness parameter of matter. The phase trajectory of the Lagrangian (1) on the Friedman moment sphere is shown in Fig. 1.

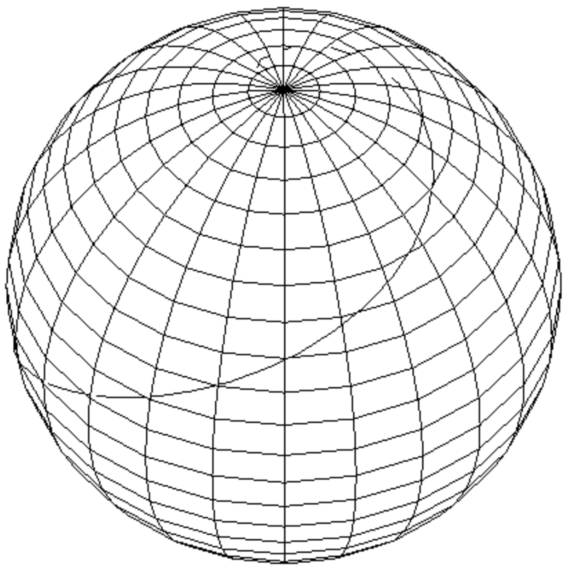

Figure 1. Loxodrome on the sphere.

The experimental values of S obtained at the Main Synchrophasotron by our colleague from the Valley of the Baobabs are shown in Table 1. In what follows,

Table 1. Some meanings S.

σ	S	σ	S	σ	S	$\approx S$
1	1	3	3	5	6	5.9909
2	2	4	4	5	6	6.0010

however, we will consider a more complex model of dark matter regeneration from the interbranch gap taking into account the interaction of superstrings with microgeons. In this case, you need to use a more complex expression than (1):

$$\mathbf{A} = \int\limits_0^\infty \frac{\xi^2 d\xi}{\sqrt{\eta+\xi^2}} \times \left(\frac{1}{e^{\sigma\xi}-1} - \frac{1}{e^{\sigma\sqrt{\eta+\xi^2}}-1} \right) \equiv$$

$$\equiv \int\limits_0^\infty \frac{\xi^2 d\xi}{\sqrt{\eta+\xi^2}} \frac{1}{e^{\sigma\xi}-1} - \int\limits_0^\infty \frac{\xi^2 d\xi}{\sqrt{\eta+\xi^2}} \frac{1}{e^{\sigma\sqrt{\eta+\xi^2}}-1}. \tag{2}$$

Conclusion

We believe that the observed phenomenon of resonant transparency is the first example of the manifestation of a nontrivial excitation of a non-emitting charge-current, and toroidal metamagnetic matrices operating at transparency resonance can serve as a source of the uncompensated vector potential **A**, although in our case the destructive interference between electric and toroidal dipole moments does not lead to a complete reduction of their scattering, the residual dipole radiation (which converges forward and backward for an infinite array of subwavelength periodicity) is suppressed by higher multipoles, while the gauge-invariant vector potential must be generated by mutually compensated contributions P and T. Here, due to the two-dimensional periodicity of the metamaterial arrays, the nontrivial component of the vector potential field is assumed to be localized in the plane of the metamaterial structure.

In conclusion, we have identified a class of metamaterials that support a novel mechanism of resonant electromagnetic transparency, which differs from the Fano noise commonly found in plasmonic systems and metamaterials that mimic electromagnetic-induced transparency. It uses destructive interference between spatially and spectrally aligned and coherently oscillating induced electric and toroidal dipoles and creates very narrow and characteristically symmetric Lorentz lines of transparency with Q factors exceeding 300. This mode of resonant excitation of the metamaterial corresponds to the long-awaited implementation of a nontrivial non-emitting charge-current configuration, which can generate waves of a gauge-irreducible vector potential in the absence of scattered (reflected) electromagnetic fields.

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Название статьи

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Аннотация. Взаимодействие сильно резонансных взаимодействий позволяет значительно улучшить функциональность многих электромагнитных устройств. Однако резонансы могут быть ослаблены джоулевыми и радиационными потерями. Хотя во многих случаях потери Джоуля могут быть сведены к минимуму благодаря выбору составляющих материалов, управление радиационными потерями часто является большой проблемой. Недавние решения включают использование связанных лучистых и сублучистых мод, дающих узкие асимметричные резонансы Фано в широком диапазоне систем: от дефектных состояний в фотонных кристаллах и оптических волноводов с мезоскопическими кольцевыми резонаторами до наноразмерных плазмонных и метамагнитных систем, проявляющих интерференционные эффекты, сходные с электромагнитным индуцированная прозрачность. Здесь мы теоретически демонстрируем и экспериментально подтверждаем новый механизм резонансной электромагнитной прозрачности, который дает очень узкие изолированные симметричные линии Лоренца в тороидальных метаматериалах.

Ключевые слова: оптика, оптический волновод, тороидальные метаматериалы

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